AD/A-001 238

AERODYNAMICS OF IN-TUBE LAUNCH PHASE OF ROCKETS (INTERNAL BALLISTICS ANALYSIS AND MINI-COMPUTER PROGRAM DEVELOPMENT)

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Prepared for:

Army Research Office

September 1974

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER UILU ENG 74-4006	2. GOV'T ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER AD/A-001 238	
4. TITLE (and Subtitle) AERODYNAMICS OF IN-TUBE LAUNCH PHASE OF ROCKETS (INTERNAL BALLISTICS ANALYSIS AND MINI-COMPUTER DEVELOPMENT)	5. TYPE OF REPORT & PERIOD COVERED Final, 1 November 1973-30 June 1974
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Helmut H. Korst	8. CONTRACT OR GRANT NUMBER(s) DAHC04-74-G-0057
9. PERFORMING ORGANIZATION NAME AND ADDRESS Gas Dynamics Laboratory, Department of Mech. & Ind. Eng., University of Ill. at Urbana-Champaign Urbana, IL 61801	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Post LRCP-U.S.A.R.O.
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army U.S.A.R.O., Box CM, Duke Station Durham, NC 27706	12. REPORT DATE
	13. NUMBER OF PAGES Thirty-Three (33)
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) U.S.A. Missile Command Redstone Arsenal, AL 35809	15. SECURITY CLASS. (of this report)
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

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INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Internal Ballistics
Rocket Launch
Mini-Computer Programs

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Aerodynamic phenomena affecting the launch phase of in-tube launched rockets are investigated. Of special interest is the motion of the missile relative to the tube before and during exiting as it results from the burning characteristics of the propellant with given grain geometry, structural constraints such as closure failure, and tube wall proximity.

(continued)

20. ABSTRACT (continued)

Theoretical analysis, based on certain simplifying assumptions such as uniform, only pressure dependent grain burning, one-dimensional quasi-steady nozzle flow, and constant flame temperature, results in systems of ordinary non-linear differential equations which have been programmed for mini-computer use.

Quantitative results are obtained which show good agreement with available data and describe the motion of the rocket during launch phase in sufficient detail. Rocket chamber pressure and nozzle thrust are evaluated over the entire duration of powered flight.

An experimental program in support of the continuing effort has been initiated by adding high-pressure capability to an existing blow-down facility at the Gas Dynamics Laboratory of the University of Illinois at Urbana-Champaign.

1. FOREWORD

Work conducted under this grant pursued two long range objectives, namely:

1. To investigate aerodynamic phenomena affecting the launch phase of in-tube launched rockets, and
2. To achieve direct compatability between in-house computer oriented (MICOM) efforts and the research conducted at the University of Illinois. This was to be achieved by acquisition of an HP 9830 system to be located in the Gas Dynamics Laboratory of the University of Illinois.

An immediate goal consisted of generating a comprehensive, yet well manageable analysis (including computer program development and typical performance documentation) of the launch and flight performance of a rocket with specified motor design and grain configuration.

Dr. H. H. Korst, Professor of Mechanical Engineering, acted as Project Director; Dr. R. A. White, Professor of Mechanical Engineering, supported by Mr. Dean H. Keal, was in charge of the facility development in the Mechanical Engineering Laboratory in preparation of the experimental effort which is to continue.

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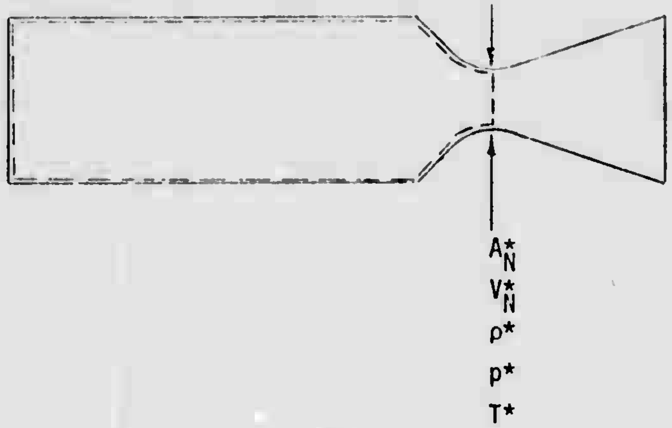

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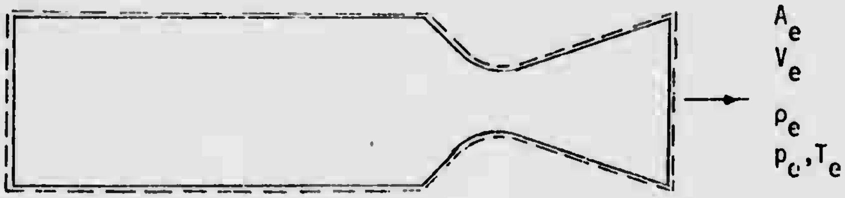

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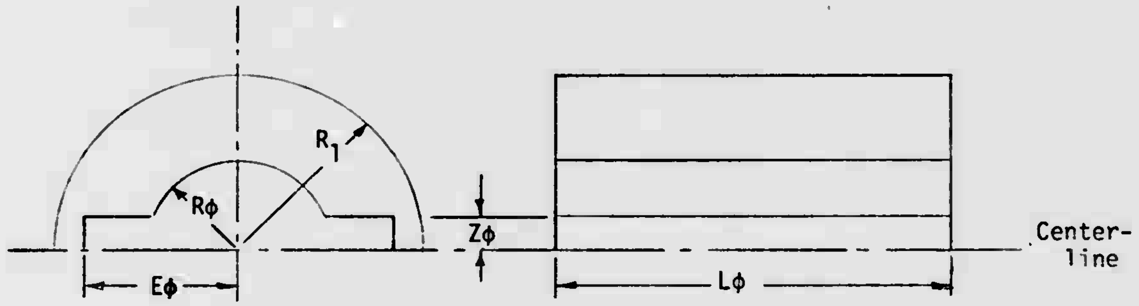

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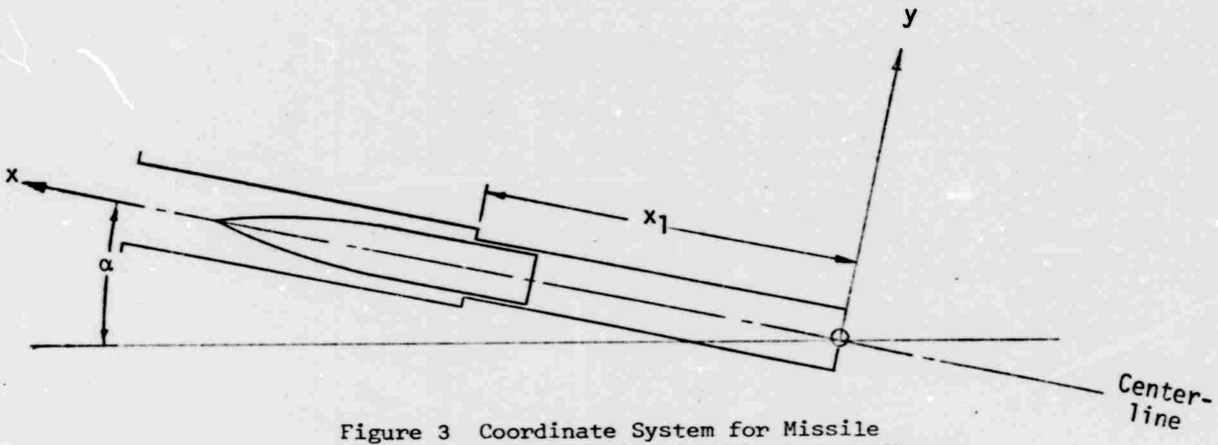

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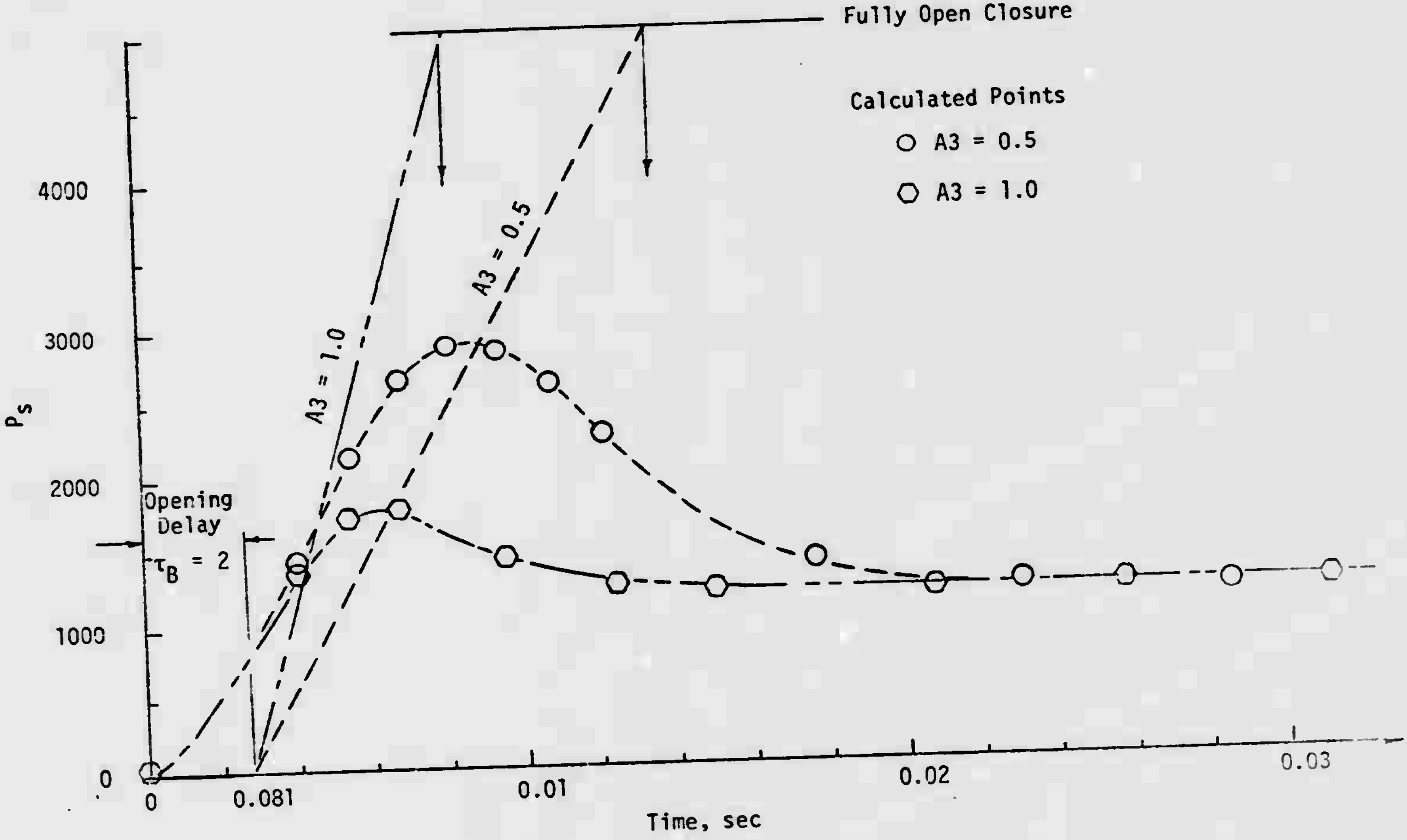

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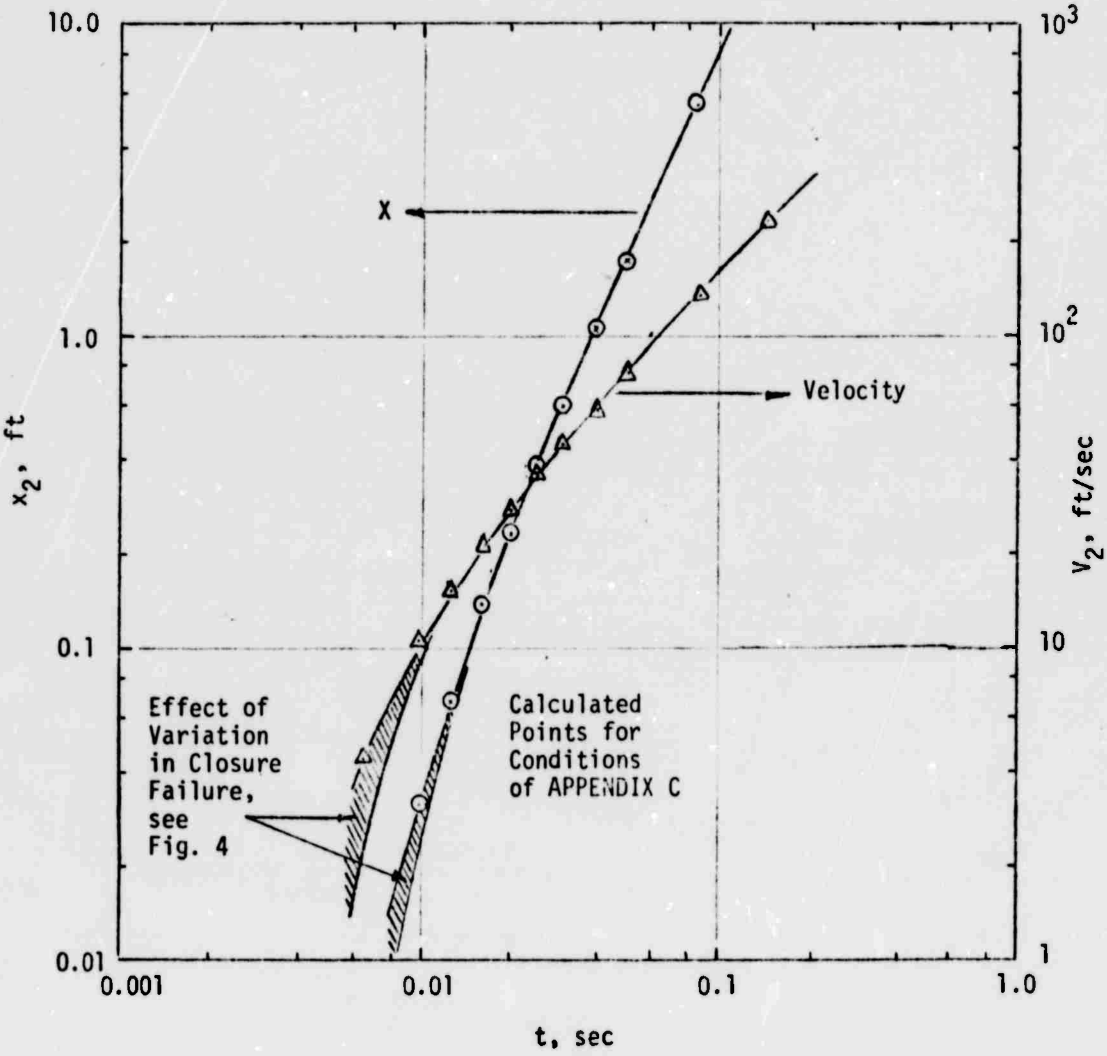

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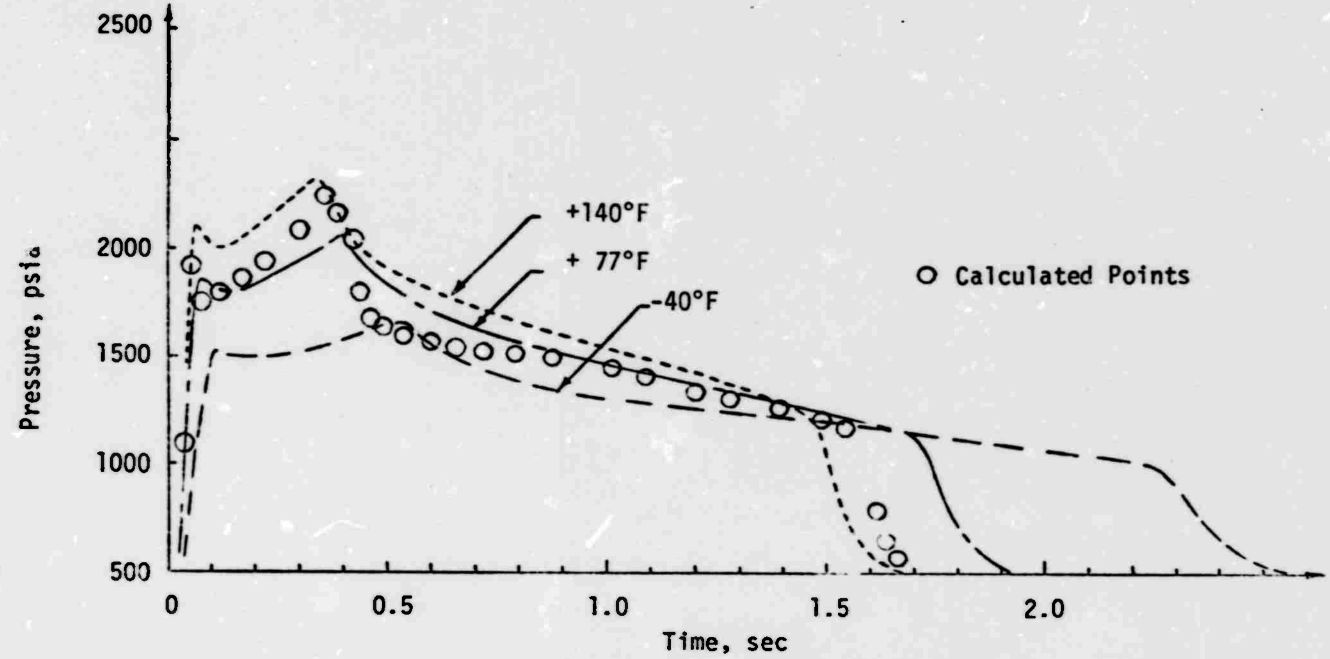

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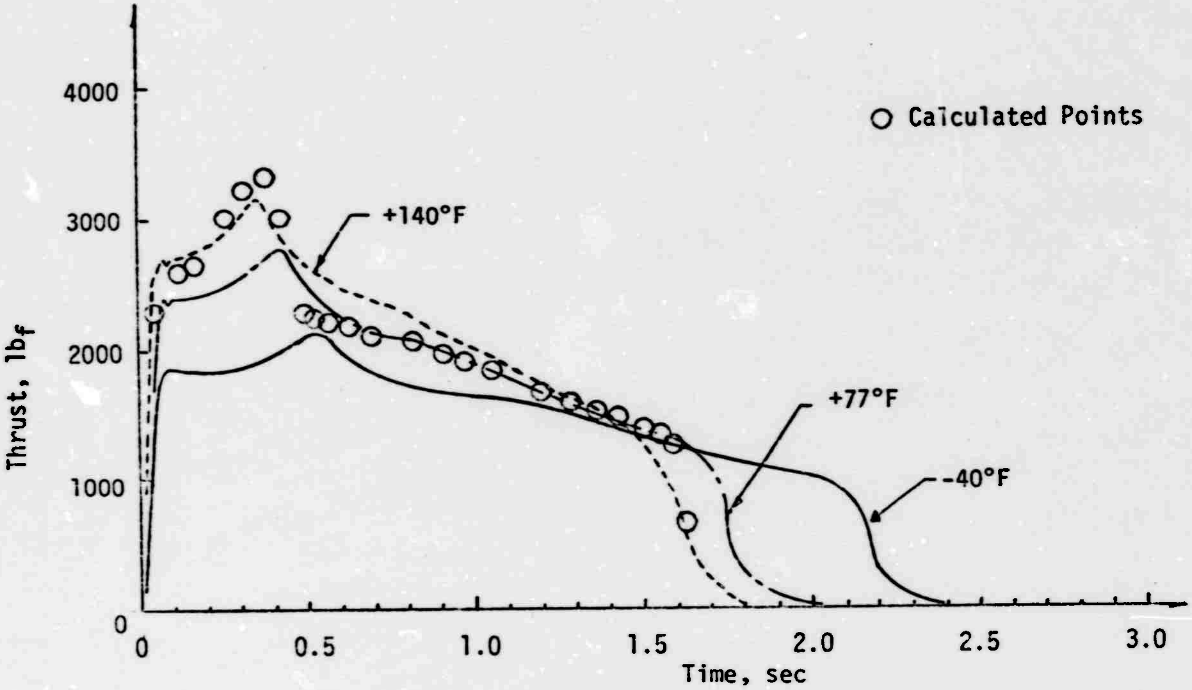

Figure 7 Thrust-Time History for ARROW Rocket

Table 1 Parameters Affecting Internal Ballistic Performance

γ	Specific heat ratio, (-)
M	Molecular weight, lb_m/mole
T_{oF}	Flame temperature, °R
v_c	Co-volume, ft^3/lb_m
v_s	Specific volume, ft^3/lb_m
B	Burning coefficient, Eq. (17)
n	Burning exponent, (-), Eq. (17)
	R1, Rϕ, Eϕ, Zϕ, Lϕ, grain dimensions, ft, see Fig. 2
W_o	Initial weight of round
L	Loading ratio, lb_m/ft^3
V_c	Chamber volume, ft^3
A_N^*	Nozzle throat area, ft^2
A_e	Nozzle exit area, ft^2
$P_{o,o}$	Reference (atm) pressure, lb_f/ft^2
π_B	Burst pressure ratio, (-)
a	Effective linear opening coefficient, (-)

4. PERFORMANCE ANALYSIS OF A DOGLEG-GRAIN, TUBE LAUNCHED ROCKET AND AUXILIARY EFFORTS

During the launch phase, as grain burning is initiated, the propulsive nozzle remains initially closed to promote faster pressure buildup. Subsequently, the closure fails, starting at a prescribed burst pressure and allowing (critical) outflow through a (linearly) time variable effective throat. Neglecting fast transients, quasi-steady outflow conditions (together with the burning law and time varying grain surface) will determine the pressure and temperature history in the combustion chamber in which the gaseous phase of the propellant is assumed to be of uniform (stagnation) state. For fully established nozzle flow conditions (the initial shock system being expelled quicly), it is then possible to determine propulsive thrust, rocket acceleration velocity, and position as functions of time. Parameters considered for the present analysis are listed in Table 1. To investigate rocket-launcher interaction, it is then also of importance to investigate the effects of plume-wall interactions and the flow of propellant gases in the launch tube.

While analytical work on the propulsive rocket motion has been completed, certain experimental phases of rocket-gas interactions within the launch tube are still in progress and will be reported on at the conclusion of a follow-up effort under ARO-sponsorship.

4.1 CONTROL VOLUME ANALYSIS OF ROCKET MOTOR PERFORMANCE

Boundaries of the control volume are selected to include the entire missile including the propulsive nozzle (Fig. 1a); however, analysis of the chamber pressure as based on the conservation of mass utilizes the somewhat smaller volume terminated by the nozzle throat (Fig. 1b). These, together with the assumption of a uniform state in the combustion chamber,

appear to be allowable simplifications [1].*

4.1.1 Conservation of Mass

With m_s and m_g denoting the mass of solid and gaseous phases of propellant in the chamber having the v-lume V_c, one introduces the specific volumes to express

$$m_g\, v_g + m_s\, v_s = V_c \tag{1}$$

and, by differentiation with respect to time, as $dv_s/dt = 0$,

$$v_s \frac{dm_s}{dt} + m_g \frac{dv_g}{dt} + v_g \frac{dm_g}{dt} = 0 \tag{2}$$

where $m_g = (V_c - v_s\, m_s)/v_g$ and

$$v_g = v_c + \frac{R\, T_o}{p_o} \tag{3}$$

(Clausius-type gas).

With $T_o = T_{oF}$ and v_c = constant

$$\frac{dv_g}{dt} = - \frac{R\, T_{oF}}{p_o^2} \frac{dp_o}{dt} . \tag{4}$$

The original mass of solid propellant at $t = 0$, m_{so}, is used to form the dimensionless ratio

$$\mu = \frac{m_s}{m_{so}} \tag{5}$$

and accounting for the outflow of gas from the control volume through the nozzle throat under choking conditions

$$\frac{dm_g}{dt} = - \frac{V_N^* A_N^*}{v_N^*} \frac{dm_s}{dt} \tag{6}$$

For convenience, we now introduce an "ideal" acoustic reference velocity

---

*Numbers in brackets refer to entries in REFERENCES.

$$C_{oR} = \sqrt{\gamma R g_c T_{oF}} \tag{7}$$

and a characteristic time

$$t_o = \frac{V_c}{A_N^* C_{oR}} \tag{8}$$

to define additional dimensionless variables,

$$\tau = \frac{t}{t_o} \tag{9}$$

$$\pi = \frac{p_o}{p_{o,o}} \tag{10}$$

$$\phi_N = \frac{V_N}{C_{oR}} \tag{11}$$

and parameters

$$C_1 = \frac{R T_{oF}}{p_{o,o} v_s} \tag{12}$$

$$C_2 = \frac{V_c}{m_{so} v_s} \frac{1}{L v_s} \tag{13}$$

Thus, the conservation of mass expressed by Eq. (2) attains the form

$$\frac{d\mu}{d\tau}\left[1 - \frac{v_c}{v_s} - \frac{C_1}{\pi}\right] - C_2\left[\frac{v_c}{v_s} + \frac{C_1}{\pi}\right] \phi_N^*\left[\frac{v_s}{v_N^*}\frac{A_N^*(\tau)}{A_N^*}\right] = \frac{C_2 - \mu}{\frac{v_c}{v_s} + \frac{C_1}{\pi}} C_1 \frac{1}{\pi^2}\frac{d\pi}{d\tau} \tag{14}$$

For nozzle flow under choked conditions, by combining Eq. (3) with Bernoulli and energy equation, we obtain $A_N^*(\tau)$

$$\phi_N^* \frac{v_s}{v_N^*} \frac{A_N^*(\tau)}{A_N^*} = \left(\frac{A_N^*(\tau)}{A_N^*}\right) \frac{\sqrt{\frac{2}{\gamma - 1}\left(1 - \frac{T^*}{T_{oF}}\right)}}{\frac{v_c}{v_s} + \frac{C_1}{\pi\left(\frac{T^*}{T_{oF}}\right)^{1/(\gamma-1)}}} \tag{15}$$

where

$$\frac{T^*}{T_{oF}} = \left\{\frac{\gamma - 1}{2}\left[1 + \frac{v_c}{v_s}\frac{\pi}{C_1}\left(\frac{T^*}{T_{oF}}\right)^{1/(\gamma-1)}\right] + 1\right\}^{-1} \tag{16}$$

Attention is now given to the opening (failure) characteristics of the closure. The nozzle is originally closed by a diaphragm which leads to a delay in the outflow through the nozzle, also promoting the initial pressure buildup in the combustion chamber. The effect of burst pressure and opening coefficients on recoilless gun operation has been discussed in greater detail [1] where also experimental evidence is presented. We are retaining here this analytical concept by introducing a burst pressure ratio π_B at which the closure begins to fail at the time τ_B in such a way that it opens the throat area linearly with time at a rate determined by the opening coefficient, A_3.

$$1 > \frac{A_N^*(\tau)}{A_N^*} = \begin{array}{l} 0 \text{ for } \tau < \tau_B \\ A_3\,(\tau - \tau_B) \text{ for } \tau > \tau_B \end{array}$$

Full opening is attained at time τ_1 where

$$\frac{A_N^*(\tau)}{A_N^*} = 1$$

after which time $A_N^*(\tau)/A_N^* \equiv 1$. It is of interest to note that the failure characteristics of the closure exert some influence on the initial phases of the rocket launch especially on the peak chamber pressure, but seem to attenuate rather quickly after full nozzle aperture has been attained.

4.1.2 Propellant Gas Generation

Expressing the burning rate in conventional form by

$$r = B\,p^{n}(\text{in/sec}) \tag{17}$$

accounting for the temperature dependency in B and the pressure dependency by the exponent n, we can determine the rate of propellant gas generation from

$$-\frac{dm_s}{dt} = \frac{r\,S(t)}{v_s\,12} \tag{18}$$

where S(t) is the time dependent burning surface of the grain, so that in dimensionless form

$$\frac{d\mu}{d\tau} = \frac{dm_s}{dt}\,\frac{d}{dm_s}\,\frac{dt}{d\tau} = \frac{S\;r\;t_o}{v_s\;m_{so}\;12} \tag{19}$$

For the initial grain configuration given in Fig. 2, a solely time dependent burning rate (note that the pressure p is assumed to be uniform over the entire surface S), will produce changes in the grain geometry so that

$$R(\tau) = R_o + \frac{t_o}{12}\int_0^{\tau} r\,d\tau$$

$$E(\tau) = E_o + \frac{t_o}{12}\int_0^{\tau} r\,d\tau \tag{20}$$

$$Z(\tau) = Z_o + \frac{t_o}{12}\int_0^{\tau} r\,d\tau$$

The burning surface S(τ) can now be expressed as follows:

(i) For $Z^2 + E^2 \leq R1^2$, one obtains

$$S = 4L\{R(90 - \theta)\,\pi/180 + E + Z - R\cos\theta] \qquad (21.1)$$

where $\theta = \sin^{-1}(Z/R)$.

(ii)
$$S = 4L\left[R(90 - \theta)\,\pi/180 + \delta R_1\sqrt{1 - (E/R1)^2} + R_1\sqrt{1 - (Z/R_1)} - R\cos\theta\right]$$

where $\delta = 1$ for $E < R1$ (21.2)

$= 0$ for $E > R1$ (21.3)

Also, for the grain configurations, Fig. 2,

$$m_{so}\, v_s = L\, R_1^2 \left\{\pi\left[1 - \left(\frac{R_o}{R1}\right)^2\right]\left[1 - \frac{\theta_o}{90}\right] + \left(\frac{R_o}{R_1}\right)^2 \sin 2\theta_o - 4R_o\, E_o \sin\theta_o\right\} \qquad (22)$$

where $\theta_o = \sin^{-1}(Z_o/R_o)$

4.1.3 Chamber Pressure-Time History

Noting that

$$\frac{r\, t_o}{12} = \frac{B\, p_{o,o}{}^n\, \pi^n\, V_c}{12\, A_N^*\, C_{oR}} \qquad (23)$$

It is possible to calculate the pressure-time history in the combustion chamber by integrating Eq. (14) together with Eq. (19) once the grain geometry (Eqs. (21) and (22)) and the burning law (Eq. (23)) are specified.*

4.2 ROCKET DYNAMICS AND KINEMATICS

4.2.1 Initial Phase

Considering the rocket motion here as resulting from nozzle thrust and the gravitational acceleration only and using a Cartesian system of coordinates aligned with the launch tube axis (see Fig. 3) which

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*A summary of parameters introduced into the present analysis is given in Table 1.

is inclined to the horizontal by the angle α, one obtains

$$\frac{dV_x}{dt}\frac{W}{g_c} = T - W\frac{g}{g_c}\sin\alpha \qquad (24)$$

and

$$\frac{dV_y}{dt}\frac{W}{g_c} = -W\frac{g}{g_c}\cos\alpha \qquad (25)$$

In restricting ourselves to these expressions, we have, for the time being, neglected all aerodynamic forces acting on the rocket. We note, however, that the lateral motion of the rocket which begins after the rocket has already moved the distance x_1 at time t_1 in the tube, may well give rise to unbalanced aerodynamic interference forces which can lead to pitching during launch.

For the initial phases of the lateral motion, we may neglect changes of the rocket mass W and, using an average thrust $\overline{T}$, integrate Eq. (25) twice to yield, as $y = 0$ and $V_y = 0$ at $x = x_1$, $t = t_1$.

$$y = -\frac{\cos\alpha\; x_1}{\frac{\overline{T}\,g_c}{W\,g} - \sin\alpha}\left(\frac{t}{t_1} - 1\right)^2 \qquad (26)$$

The time t_1 where the distance x_1 has been negotiated is found from the integration of Eq. (24) for which $V_x = 0$ at $t = 0$ and

$$x = g\,\frac{t^2}{2}\left(\frac{\overline{T}\,g_c}{W\,g} - \sin\alpha\right) \qquad (27)$$

so that

$$t_1 = \frac{2x_1}{\frac{\overline{T}\,g_c}{W} - g\sin\alpha} \qquad (28)$$

4.2.2 Effect of Variable Thrust and Mass

If the full trajectory is to be followed, it will be necessary to introduce aerodynamic lift and drag forces and to consider also the time variations of rocket thrust and mass. Accounting for the latter two only, one realizes the results of pressure chamber and propellant mass ratio histories as they result from Section 4.1.

While Fig. 4 illustrates the influence of closure failure on the early chamber pressure history, Fig. 5 shows that the effect on launch velocity is indeed very small.

The full chamber pressure history, as calculated by the present program (for instantaneous closure failure), is compared to available ARROW data in Fig. 6.

Accounting for the variable rocket mass by

$$W = W_o \left[1 - \frac{ms_o}{W_o} (1 - \mu) \right] \qquad (29)$$

and determining the thrust force on the basis of the control volume shown in Fig. 1a, the time-dependent thrust force is given (in lb_f) by

$$T = p_{o,o} A^*_N \, 144 \left[\frac{2\gamma}{\gamma - 1} \frac{\sqrt{\left(1 - \frac{T_e}{T_{oF}}\right)\left(1 - \frac{T^*}{T_{oF}}\right)}}{\left(\frac{v_c}{v_s} \frac{1}{C_1} + \frac{1}{\pi} \frac{T^*}{T_{oF}}\right)^{1/(1-\gamma)}} + \frac{A_e}{A^*_N} \left[\pi \left(\frac{T_e}{T_{oF}} \right)^{\gamma/(\gamma-1)} - 1 \right] \right] \qquad (30)$$

Again, the thrust force calculated with the present program is compared with data provided for the ARROW rocket (see Fig. 7). The temperature ratio T_e/T_{oF} is found after solving Eq. (16) for T^*/T_{oF} by iteration, from

$$\frac{T_e}{T_{oF}} = \left\{ \left[\frac{\pi}{C_1} \frac{v_e}{v_s} + \left(\frac{T^*}{T_{oF}} \right)^{1/(1-\gamma)} \right] \sqrt{\frac{(1 - T_e/T_{oF})}{(1 - T^*/T_{oF})}} \frac{A_e}{A_N^*} - \frac{\pi}{C_1} \frac{v_e}{v_s} \right\}^{(1-\gamma)} \qquad (31)$$

which also requires iteration.

4.3 COMPUTER PROGRAM DEVELOPMENT

As already pointed out in Section 1, the broader objectives of this grant called for computer program compatibility between MICOM and the supporting effects at the University of Illinois. Consequently, after acquisition of the HP 9830 system, a considerable number of computer programs have been generated which are now operational at both locations. Some representative examples are cited in the following

4.3.1 Internal Ballistics Program

4.3.1.1 <u>Bomb, Gun, and Recoilless Gun Performance Analysis</u>

(Documented earlier [1].)

4.3.1.2 <u>Adaptations of such Programs to Deal with Gun-Launched Rockets [2]</u>

4.3.2 Plume and Slipstream Boundary Analysis based on the Method of Successive Centered Expansions [3,4]

4.3.3 Base Pressure Analysis for Unpowered Flight of Rockets and Projectiles [5,6]

4.3.4 Viscous Jet Mixing and Boundary Layer Programs in Support of Drag Evaluations for Missiles [7]

4.3.5 Shock Interaction Programs Applicable to Muzzle Break Blast-wave Propagation and Reflections [1]

4.3.6 Comprehensive Program for the DOGBONE Grain (ARROW) Rocket Performance (based on the analysis of this report)

4.3.6.1 Program Printout

A printout of the program for the HP 9830 computer is given in APPENDIX A.

4.3.6.2 User's Instructions

User's instructions are given in APPENDIX B with all input quantities defined (also, see Table 1).

4.3.6.3 Program Output ("PRINT ALL" Mode)

Program output in the "PRINT ALL" mode of the comuter is given in APPENDIX C for selected input data as listed. It must be noted that the DOGLEG GRAIN geometry (R1, R0, E0, Z0) and nozzle area ratio A_e/A_N^* = A5 are not entered from the keyboard but are READ by the program from the DATA line 120. Changes in these parameters have thus to be made (if so desired) by "FETCH 120, EXECUTE" and by altering the values in this line (conveniently done by using the editing features of the HP 9830).

4.4.1 Facility Development

The blow-down wind tunnel facility and auxiliary air supplies of the Mechanical Engineering Laboratory, Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, have been modified to allow modeling of the tube launch system under quasi-steady operating conditions. The pressure distribution within the launch tube due to plume-tube wall interactions and vehicle eccentricity is the primary objective.

4.4.1.1 Air Supply Modifications

To provide for representative modeling, it was necessary to modify the existing auxiliary air supply system to accept the higher pressures needed for simulating the rocket jet plume. This has been accomplished by the installation of new high pressure storage (approximately 450 ft^3, maximum working pressure 1800 psig) and high pressure piping in compliance with OSHA regulations. A two-stage compressor allowing pumping of the system to either 250 psig or 500 psig levels is currently being utilized.

4.4.1.2 Model Constructed

A one-half scale geometrically similar model has been constructed of both the afterbody and the launch tube system. The model consists of an interchangeable nozzle section installed in the end of a section of schedule 80 high pressure pipe with nominal outside diameter of 1.90 inches. The launch tube is simulated by two lengths of thin wall brass tubing with nominal inside diameters of 2.0 and 2.25 inches.

4.4.2 Preliminary Experiments Conducted

A series of calibration checks with the nozzle configuration selected were carried out to determine the performance characteristics of the modified high pressure system and its control valve. The results of the preliminary tests indicated that maximum stagnation pressures of only 180 psig are presently reached with tank pressures of approximately 450 psig. This is below the level desired for proper plume simulation within the launch tube. It appears that the flow level capability of the control valve is too close to the nozzle mass flow required or that a sonic throat

is occurring within the pipe system restricting the available pressure to the nozzle. Additional tests are being planned to determine and subsequently improve the range of system performance. A reduction in model scale to approximately 0.25 may be required.

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APPENDIX

Computer Program "[illegible] GRAIN ROCKET"
Printout for HP 9830 4-K (no [illegible])

```
LIST

5 PRINT "[illegible]"
10 DEF FNS(X)=[illegible]
15 FLOAT 4
20 DEG
25 DISP "GAMMA=";
30 INPUT K
35 DISP "MOLECULAR WEIGHT PROP.=";
40 INPUT M
45 DISP "FLAME TEMP, DEG R=";
50 INPUT T0
55 C0=(K*32.174*1545*T0/M)↑0.5
60 DISP "COVOLUME PROP.VC=";
65 INPUT V1
70 DISP "CHAMBER VOL. V=";
75 INPUT V
80 DISP "SPEC. VOL. SOLID, VS=";
85 INPUT V2
90 DISP "LOAD RATIO,L=";
95 INPUT L
100 DISP "INITIAL MASS, MISSILE";
105 INPUT M0
110 C2=1/(L*V2)
115 READ R1,R0,E0,Z0,A5,L0
120 DATA 0.14167,0.057292,0.115,0.015625,3.7600,2.5938
125 PRINT "R1="R1;"R0="R0;"E0="E0;"Z0="Z0;"AE/A*="A5
130 DISP "BURN. COEFF. B=";
135 INPUT B
140 DISP "BURN. EXP.N=";
145 INPUT N
150 DISP " BURST PRESS. RATIO =";
155 INPUT B2
160 DISP "AREA NOZZLE, A1=";
165 INPUT A1
170 DISP "P0,REF=";
175 INPUT P0
180 T9=V/(A1*C0)
185 C1=1545*T0/(144*P0*V2*M)
190 DISP "OPENING COEFF.A=";
195 INPUT A3
200 DISP "TIME INCREM.(1)=";
205 INPUT T
210 PRINT "GAMMA="K;"M="M;"T0="T0;"COVOL VC="V1;"CHAMB.VOL.V="V
215 PRINT "LOAD RATIO L="L;"SPEC. VOL.SOL.VS="V2;"INITIAL MASS, MISS.="M0
220 PRINT "BURN COEF.B="B;"BURN EXP.N="N
225 PRINT "BURST PLATE PRESS.="B2
230 PRINT "AREA NOZZLE A1="A1;"REF PRESS. P0="P0;"OPEN. COEFF.ALPHA="A3
235 PRINT
240 V3=V1/V2
245 D0=FNS(Z0/R0)
250 B1=B*(P0)↑N
255 D3=PI*(1-(R0/R1)↑2*(1-D0/90))+(R0/R1)↑2*(TN(2*D0)-4*E0*R0*SIND0/R1↑2
260 D1=-B1*T9/(3*R1*D3)
```

```
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
295 Z=[illegible]
300 E=1.0
305 A4=0
310 T1=0
315 M1=0
320 X1=0
325 I=1
330 G3=1
335 P9=P1
340 D=FN[illegible]
345 IF ((R*SIN(D))↑2[illegible] THEN 625
350 B9=D1*P9*[illegible](180*PI)*L R1-COS([illegible] R1*2/R1)
355 A=1
360 U=(((1+V3*P5*(A[illegible]*2*1)↑(-1)
365 IF ABS(A-U)<0.0001 THEN 380
370 A=U
375 GOTO 360
380 H=(2*(1-U)*(K-1)[illegible]0.5*V3*C1*U↑(1/(1-K))/P0)
385 IF G1=1 THEN 400
390 IF P2>P1 THEN 410
395 IF G3=1 THEN 625
400 A4=A3*(T1-T0)
405 IF A4>1 THEN 645
410 M2=M1+B9*T
415 Y1=T*P9*(P5*V3*C[illegible]1)/(2*(M1+M2)/P9)
420 Y2=B9*(1-V3*C1/P9)
425 Y3=-C2*(V3+C1/P9)
430 Y4=Y3+A4*H
435 P2=P1+Y1*(Y2+Y4)
440 P9=(P2+P1)/2
445 Q2=0.55
450 Q1=((P9*V3*C1+Q↑(1/(1-K)))*((1-Q2)/(1-Q))↑0.5*A5-P9*V3*C1)↑(1-K)
455 IF ABS(Q1-Q2)<0.001 THEN 470
460 Q2=Q1
465 GOTO 450
470 F1=2*K*((1-Q1)*(1-Q))↑0.5/((K-1)*(V3/C1+Q↑(1/(1-K))/P9))
475 F2=A5*(P9*Q1↑(1/(K-1))-1)
480 F3=P0*A1*(F1+F2)*144
485 V2=V1+A4*F3*T*T9*32.174/(M0*(1-(L0*R1↑2+B3*(V2*M2[illegible]*(1/(M2+M1)↑2)[illegible]
490 X2=X1+(V2+V1)*T*T9/2
495 IF G3=1 THEN 505
500 GOTO 515
505 G3=0
510 GOTO 345
515 IF A4#1 THEN 525
520 IF I#10 THEN 555
525 PRINT "TAU="T1;"PRESS="P2*P0;"MOZ="M2;
530 PRINT "A2(T1)="A2;"A4";"T2(SEC)=";T1*T9
535 PRINT "THRUST (LBF)="F3;"VELOCITY (FPS)="V2;"X2(FT)="X2
540 PRINT
```

```
[illegible]
[illegible]
555 J=I+1
560 P1=P2
565 N1=N2
570 T1=T1+C
575 D2=[illegible]
580 R=R+D2
585 E=R+D2
590 Z=Z+D2
595 M1=M2
600 X1=X2
605 IF P9<1 THEN 625
610 IF M2>0 THEN 330
615 D1=0
620 GOTO 330
625 G2=0
630 T8=T1
635 G1=1
640 GOTO 400
645 A4=1
650 GOTO 410
655 F=1
660 IF R1<E THEN 670
665 F=0
670 B8=(R+(90-D)*PI/180*R1-F*COS(D)*R1+F*[illegible]*ATN(1-(E/R1)↑2))
675 B9=D1*P9*N*(B8+SQR(1-(Z/R1)↑2))
680 GOTO 355
685 STOP
690 END
```

APPENDIX B

User's Instructions for "DOGLEG GRAIN ROCKET" Program

ARROW-BALLISTIC PROGRAM

HP 9830 (no ROMS needed) 690/1428 "DOGBONE GRAIN ROCKET" load

Input (asked for by program)	Symbol
1. Gamma (propellant), 1	K
2. Molecular weight propellant, 1	M
3. Flame temperature, propellant, °R	T0
4. Co-volume, propellant gas, ft^3/lb_m	V1
5. Combustion chamber, volume, ft^3	V
6. Specific volume, solid propellant, ft^3/lb_m	V2
7. Load ratio, lb_m/ft^3	L
8. Initial mass, missile and propellant, lb_m	M0
(location 120 generates (can be altered) Data R1, R0, E0, Z0, A5 = A_e/A_N^* and L0 related to the DOGBONE geometry and nozzle configuration as shown in Figs. 1a, 1b, and 2)	
9. Burning coefficient, in./sec	B
10. Burning law, exponent, 1	N
11. Burst pressure ratio, 1	B2
12. Area nozzle (throat A^*), ft^2	A1
13. Reference pressure P0, psia	P0
14. Opening coefficient (burst plate), 1	A3
15. Time increment ($\Delta\tau$), 1	T

NOTE: It is suggested to use T = 0.5 for accuracy and numerical stability. Computer program, after nozzle throat has fully opened (after fast transients of burst plate opening have been recorded) skips intermediate PRINTOUTS (retaining T), gives data for only every tenth time increment. This will generate one set of data every 100 seconds (approximate real computer time). Program stops when combustion chamber pressure reaches P0.

APPENDIX C

Program Output

```
RUN
[illegible]
GAMMA=1.1[illegible]
MOLECULAR WEIGHT M=[illegible]
FLAME TEMP. DEG [illegible]
COVOLUME PROP. VC=[illegible]
CHAMBER VOL. V=[illegible]
SPEC. VOL. SOLID. VS=[illegible]
LOAD RATIO L=[illegible]
INITIAL MASS, MISSILE=47.[illegible]
A1= 1.4167E-01    A0= 5.[illegible]    [illegible]    D= 1.5[illegible]
ALPHA= 3.7602E+00
BURN  COEFF. B=0.025
BURN. EXP.N=.45
 BURST PRESS. RATIO =10
AREA NOZZLE. A1=.00[illegible]
PO,REF=14.4
OPENING COEFF.A=1
TIME  INCREM.(1)=.1
GAMMA= 1.183[illegible]E+00    M= 2.265[illegible]E+01    T0= [illegible]    COVOL VC=
 2.1460E-02    CHAMB.VOL V= 1.6[illegible]
LOAD RATIO L= 8.4500E+01    SPEC. VOL.SOL VS= [illegible]
INITIAL MASS, MISS = 4.7[illegible]E+01
BURN COEF.B= 2.5000E-02    BURN EXP.N= 4.5000E-01
BURST PLATE PRESS = 1.0000E+01
AREA NOZZLE A1= 6.9906E-0[illegible]    REF PRESS  PO= 1.4400E+01
OPEN. COEFF.ALPHA= 1.0000E+00

TAU1= 0.0000E+00    PRESS= 8.0[illegible]E+01    MU2= 1.9993E-01    A2(T1)/A2=
 0.0000E+00    T2(SEC)= 5.4393E-04
THRUST (LBF)= 2.6354E+01    VELOCITY (FPS)= 0.0000E+00    X2(FT)=
 0.0000E+00

TAU1= 1.0000E-01    PRESS= 1.9956E+02    MU2= 9.9979E-01    A2(T1)/A2=
 0.0000E+00    T2(SEC)= 1.0879E-03
THRUST (LBF)= 2.2910E+02    VELOCITY (FPS)= 0.0000E+00    X2(FT)=
 0.0000E+00

TAU1= 2.0000E-01    PRESS= 3.6705E+02    MU2= 9.9950E-01    A2(T1)/A2=
 0.0000E+00    T2(SEC)= 1.6318E-03
THRUST (LBF)= 5.1717E+02    VELOCITY (FPS)= 0.0000E+00    X2(FT)=
 0.0000E+00

TAU1= 3.0000E-01    PRESS= 5.0917E+02    MU2= 9.9937E-01    A2(T1)/A2=
 1.0000E-01    T2(SEC)= 2.1757E-03
THRUST (LBF)= 8.9875E+02    VELOCITY (FPS)= 3.2849E-02    X2(FT)=
 8.9337E-06

TAU1= 4.0000E-01    PRESS= 7.[illegible]E+02    MU2= 9.9[illegible]E-01    A2(T1)/A2=
 2.0000E-01    T2(SEC)= 2.7[illegible]E-03
THRUST (LBF)= 1.3[illegible]E+03    VELOCITY (FPS)= 1.[illegible]E-01    X2(FT)=
 5.3249E-05
```

```
TAU1= [illegible]   PRES= [illegible]   MU2= [illegible]   A2(T1)/A2=
 [illegible]   T2(SEC)= [illegible]
THRUST (LBF)= [illegible]   VELOCITY (FPS)= [illegible]   X2(FT)=
 4.41[illegible]E-0[illegible]

TAU1= 7.0000E-01   PRES= 1.[illegible]E+03   MU2= [illegible]E-01   A2(T1)/A2=
 5.0000E-01   T2(SEC)= [illegible]15E-03
THRUST (LBF)= 2.[illegible]E+03   VELOCITY (FPS)= 1.[illegible]E+00   X2(FT)=
 9.1905E-04

TAU1= 8.0000E-01   PRES= 1.6[illegible]E+03   MU2= [illegible]E-01   A2(T1)/A2=
 6.0000E-01   T2(SEC)= 4.[illegible]E-03
THRUST (LBF)= [illegible].9500E+03   VELOCITY (FPS)= 1.[illegible]E+00   X2(FT)=
 1.6965E-03

TAU1= 9.0000E-01   PRES= 1.[illegible]E+03   MU2= [illegible]E-01   A2(T1)/A2=
 7.0000E-01   T2(SEC)= 5.4[illegible]E-03
THRUST (LBF)= 3.[illegible]427E+03   VELOCITY (FPS)= 3.1933E+00   X2(FT)=
 2.8500E-03

TAU1= 1.0000E+00   PRES= 1.5[illegible]E+03   MU2= 5.[illegible]677E-01   A2(T1)/A2=
 8.0000E-01   T2(SEC)= 5.99[illegible]E-03
THRUST (LBF)= 3.1347E+03   VELOCITY (FPS)= [illegible].4365E+00   X2(FT)=
 4.4748E-03

TAU1= 1.1000E+00   PRES= 1.5450E+03   MU2= [illegible]E-01   A2(T1)/A2=
 9.0000E-01   T2(SEC)= 6.5212E-03
THRUST (LBF)= 3.1220E+03   VELOCITY (FPS)= 4.4676E+00   X2(FT)=
 6.6244E-03

TAU1= 2.1000E+00   PRES= 1.2215E+03   MU2= 5.9241E-01   A2(T1)/A2=
 1.0000E+00   T2(SEC)= 1.1977E-02
THRUST (LBF)= 2.4798E+03   VELOCITY (FPS)= 1.1356E+01   X2(FT)=
 5.8820E-02

TAU1= 3.1000E+00   PRES= 1.1739E+03   MU2= [illegible]867E-01   A2(T1)/A2=
 1.0000E+00   T2(SEC)= 1.[illegible]406E-02
THRUST (LBF)= 2.3530E+03   VELOCITY (FPS)= 2.[illegible]E+01   X2(FT)=
 1.6093E-01

TAU1= 4.1000E+00   PRES= 1.1672E+03   MU2= [illegible]2495E-01   A2(T1)/A2=
 1.0000E+00   T2(SEC)= 2.2845E-02
THRUST (LBF)= 2.3543E+03   VELOCITY (FPS)= 3.1673E+01   X2(FT)=
 3.1000E-01

TAU1= 5.1000E+00   PRES= 1.1716E+03   MU2= [illegible]121E-01   A2(T1)/A2=
 1.0000E+00   T2(SEC)= [illegible]E-02
THRUST (LBF)= 2.[illegible]E+03   VELOCITY (FPS)= 4.[illegible]E+01   X2(FT)=
 5.0572E-01
```

```
[illegible]
 1.[illegible]
[illegible]
 1.[illegible]E+01

TAU1= [illegible]    PRESS= [illegible]    MU2= [illegible]    A2(IN2)=
 1.0000E+00    T2(SEC)= [illegible]
THRUST (LBF)= 2.[illegible]E+03    VELOCITY (FPS)= [illegible]    X2(FT)=
 1.0[illegible]E+00

TAU1= 8.1000E+00    PRESS= [illegible]    MU2= [illegible]    A2(IN2)=
 1.0000E+00    T2(SEC)= [illegible]
THRUST (LBF)= 2.[illegible]E+03    VELOCITY (FPS)= [illegible]    X2(FT)=
 1.3753E+00

TAU1= 9.1000E+00    PRESS= [illegible]    MU2= [illegible]    A2(IN2)=
 1.0000E+00    T2(SEC)= [illegible]
THRUST (LBF)= 2.4[illegible]E+03    VELOCITY (FPS)= [illegible]E+01    X2(FT)=
 1.7004E+00

TAU1= 1.0100E+01    PRESS= 1.2[illegible]E+03    MU2= 9.[illegible]E-01    A2(IN2)=
 1.0000E+00    T2(SEC)= 5.[illegible]E-02
THRUST (LBF)= 2.4206E+03    VELOCITY (FPS)= 8.[illegible]E+01    X2(FT)=
 2.1939E+00

TAU1= 1.1100E+01    PRESS= 1.2[illegible]E+03    MU2= 9.[illegible]E-01    A2(IN2)=
 1.0000E+00    T2(SEC)= 6.[illegible]E-02
THRUST (LBF)= 2.4[illegible]E+03    VELOCITY (FPS)= 9.[illegible]E+01    X2(FT)=
 2.6700E+00

TAU1= 1.2100E+01    PRESS= 1.2255E+03    MU2= 9.[illegible]E-01    A2(IN2)=
 1.0000E+00    T2(SEC)= 6.[illegible]E-02
THRUST (LBF)= 2.4542E+03    VELOCITY (FPS)= 1.[illegible]E+02    X2(FT)=
 3.2071E+00

TAU1= 1.3100E+01    PRESS= 1.23[illegible]E+03    MU2= 9.5045E-01    A2(IN2)=
 1.0000E+00    T2(SEC)= 7.1[illegible]E-02
THRUST (LBF)= 2.4697E+03    VELOCITY (FPS)= 1.1100E+02    X2(FT)=
 3.7871E+00

TAU1= 1.4100E+01    PRESS= 1.2[illegible]E+03    MU2= 9.[illegible]E-01    A2(IN2)=
 1.0000E+00    T2(SEC)= 7.[illegible]E-02
THRUST (LBF)= 2.4863E+03    VELOCITY (FPS)= 1.[illegible]E+02    X2(FT)=
 4.4189E+00
```